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LETTER TO THE EDITOR

Generalization of the Landau liquid concept: example of the Luttinger liquids

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Abstract. We introduce a new representation of the Bethe *ansatz* solutions for 1D Luttinger liquids which describes the spectral properties and asymptotic behaviour of the correlation functions of the 1D Hubbard model in terms of the renormalized interaction of charge and spin pseudoparticles. The study of the low-lying eigenstates is reduced to the familiar languages of band theory and of the Fermi liquid. Our results provide a better understanding of the decoupling of charge and spin degrees of freedom in 1D interacting systems, generalize the concept of a Landau liquid and may be relevant to the physics of higher dimensional systems.

The purpose of this letter is to provide evidence for the fact that the usual Landau–Fermi liquids constitute an example of a much wider class of non-trivial many-body fermionic systems (probably of all dimensions). The present study is restricted to one-dimensional (1D) Luttinger liquids [1] that are soluble by the Bethe *ansatz*, and in particular to the (1D repulsive Hubbard model. Nonetheless, we believe that many of the features found in this letter have a universal character and are also present in the low-energy physics of nearly all non-trivial many-body fermionic liquids. These common features follow essentially from the fact that the low-energy properties are fully controlled by the departure of the pseudo-momentum distribution(s) of the pseudoparticles (often many-body collective modes specific to each system) from its (their) value(s) in the interacting ground state. Moreover, after renormalization the generalized Landau liquids have only forward (or exchange) scattering. As in the Fermi liquid, the two-pseudoparticle f-functions (second functional derivatives of the energy with respect to the fluctuations) regulate the forward scattering renormalized interaction of the pseudoparticles in the low-energy regime. Our results are fully consistent with the original idea of Anderson [2] that the Luttinger liquid is a fixed point of the same renormalization group which, in some three-dimensional systems, leads to the Landau–Fermi liquid as a unique fixed point.

Although we concentrate our investigation on the case of the 1D Hubbard model, the generalization of the results to the other 1D Luttinger liquids is straightforward and will be presented elsewhere. A full understanding of the one-dimensional Hubbard model solution [3] is of interest in its own right, and may provide clues to the understanding of higher dimensional systems [2]. We consider the 1D Hubbard model

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at arbitrary magnetic field

$$\hat{H} = -t \sum_{j,\sigma} \left(c^{+}_{j\sigma} c_{j+1\sigma} + c^{+}_{j+1\sigma} c_{j} \right) + U \sum_{j} c^{+}_{j\uparrow} c_{j\uparrow} c^{+}_{j\downarrow} c_{j\downarrow} - \mu_0 H \sum_{j,\sigma} \sigma c^{+}_{j\sigma} c_{j\sigma} \tag{1}$$

where $c^{+}_{j\sigma}$ ($c_{j\sigma}$) is the creation (annihilation) operator for an electron with spin σ at site j. The system consists of N electrons on N_a sites. We make use of the following notation: the dimensionless on-site repulsion $u = U/4t$, the density $n = N/N_a$ ($k_F = \pi n/2$), the spin density $s = \left(k_{F\uparrow} - k_{F\downarrow}\right)/2\pi$, where $k_{F\sigma} = \pi N_\sigma / N_a$, and $N_\uparrow = M'$ and $N_\downarrow = M$ are the number of up and down spins in the system.

Lieb and Wu [3] used the Bethe-*ansatz* technique to reduce the eigenvalue problem for (1) to that of solving a set of coupled algebraic equations. The crystal momentum P and the energy E are given by

$$P = \sum_{j=1}^{N} q_j + \sum_{\alpha=1}^{M} p_\alpha \tag{2}$$

$$E = \sum_{j=1}^{N} \left(-2t \cos K_j \right) - 2\mu_0 N_a H s \tag{3}$$

where we make use of the notation $q_j = (2\pi/N_a)\, I_j$ and $p_\alpha = (2\pi/N_a)\, J_\alpha$. An eigenstate of the many-body system is uniquely specified by a particular choice of the quantum numbers $\{I_j\}$, $\{J_\alpha\}$ (or pseudo-momentum distributions $\{q_j\}, \{p_\alpha\}$). For instance, in the ground state I_j and J_α are consecutive integers (or half-odd integers) centred around the origin [3]. After choosing the set $\{q_j\}$, $\{p_\alpha\}$, the Lieb and Wu algebraic equations determine the charge and spin rapidities $K_j = K_j(q_j)$ and $S_\alpha = S_\alpha(p_\alpha)$, respectively ($S_\alpha = \Lambda_\alpha / u$) [3]. Although our formulation can be extended to excited states described by complex roots [4], we restrict the present study mainly to excitations involving only real rapidities.

In the thermodynamic limit ($N_a \to \infty$, n fixed) the roots $K_j = K_j(q_j)$ and $S_\alpha = S_\alpha(p_\alpha)$ proliferate on the real axis and the Lieb and Wu equations lead to [4]

$$K(q) = q + \frac{1}{\pi} \int_{-k_{F\downarrow}}^{k_{F\downarrow}} \mathrm{d}p'\, N_\downarrow(p') \tan^{-1}\left[S(p') - (1/u) \sin K(q) \right] \tag{4}$$

$$p = \frac{1}{\pi} \int_{-\pi}^{\pi} \mathrm{d}q'\, M_c(q') \tan^{-1}\left[S(p) - (1/u) \sin K(q') \right] - \frac{1}{\pi} \int_{-k_{F\downarrow}}^{k_{F\downarrow}} \mathrm{d}p'\, N_\downarrow(p') \tan^{-1}\left[\tfrac{1}{2} \left(S(p) - S(p') \right) \right]. \tag{5}$$

($K(q)$, $S(p)$ are simply related to the distributions of [3].) Moreover, the energy (3) yields

$$E = \frac{N_a}{2\pi} \int_{-\pi}^{\pi} \mathrm{d}q'\, M_c(q') \left[-2t \cos K(q') \right] - 2\mu_0 N_a H s \tag{6}$$

where

$$s = \frac{1}{2\pi} \left(\frac{1}{2} \int_{-\pi}^{\pi} \mathrm{d}q'\, M_c(q') - \int_{-k_{F\downarrow}}^{k_{F\downarrow}} \mathrm{d}p'\, N_\downarrow(p') \right). \tag{7}$$

In the right-hand sides of equations (4)–(7) $M_c(q)$ and $N_\downarrow(p)$ may be interpreted as pseudo-momentum distributions of charge and spin pseudoparticles, respectively. For eigenstates involving only real rapidities, we always have: $K(\pm\pi) = \pm\pi$ and $S(\pm k_{F\uparrow}) = \pm\infty$. Moreover, for the ground state at fixed magnetization $K(q)$ and $S(p)$ are odd functions such that $K(2k_F) = Q$ and $K(k_{F\downarrow}) = B/u$, where Q and B are the usual cut offs of the Lieb and Wu equations [3]. In this case the distributions $M_c(q)$ and $N_\downarrow(p)$ read

$$M_c^0(q) = \Theta(2k_F - |q|) \qquad N_\downarrow^0(p) = \Theta(k_{F\downarrow} - |p|)\,. \tag{8}$$

As in Fermi liquid theory these distributions do not depend on the interaction. In the present two-fluid Landau liquid the charge and spin pseudo-Fermi surfaces are defined as the set of points $\{q = \pm 2k_F,\ p = \pm k_{F\downarrow}\}$ separating the occupied from the unoccupied region. The limits of the pseudo-Brillouin zones of the charge and spin pseudoparticles are $\{q = \pm\pi\}$, $\{p = \pm k_{F\uparrow}\}$, respectively. The restrictions on the numbers I_j and J_α [3] imply that each pseudo-momentum value cannot be occupied by more than one pseudoparticle, i.e. the pseudoparticles have fermionic character [4]. These can be identified with the 'pseudo-fermions' considered in [5] and are related to the holons and spinons [2,4]. The spin pseudoparticles are of the same kind of the ones of the Heisenberg chain [4], being closely related to the spin-$\frac{1}{2}$ 'spin waves' introduced in [6]. They are many-body collective modes and in contrast to the quasiparticles of the Fermi liquid theory, which in the limit of vanishing interaction map onto real particles (electrons), the present class of pseudoparticles cannot exist outside the many-body system for any value of the bare interaction (including vanishing interaction). This feature of the Landau–Luttinger pseudoparticles [4] is related to the 'infrared catastrophe' of [2].

We consider small pseudo-momentum fluctuations around the ground-state distributions, equations (8)

$$M_c(q) = M_c^0(q) + \delta_c(q) \qquad N_\downarrow(p) = N_\downarrow^0(p) + \delta_\downarrow(p)\,. \tag{9}$$

As in Fermi liquid theory, the departure of the pseudo-momentum distribution functions of the pseudoparticles from their values in the ground state $\delta_c(q), \delta_\downarrow(p)$, equations (9), fully controls the low-energy physics of the model: This is true both for elementary excitations involving real and complex rapidities [4]. The main point in our approach is to consider $K(q)$, $S(p)$ and E (see equations (4)–(6)) as functionals of the pseudoparticle distributions. Provided that these involve a small number of pseudoparticles, an expansion of the energy $E = E_0 + E_1 + E_2 + \cdots$ can be performed to arbitrary order in the fluctuations. In the case of excitations described only by real rapidities, the leading order corrections read

$$E_1 = \frac{N_a}{2\pi}\int_{-\pi}^{\pi} \mathrm{d}q\, \delta_c(q)\epsilon_c(q) + \frac{N_a}{2\pi}\int_{-k_{F\uparrow}}^{k_{F\uparrow}} \mathrm{d}p\, \delta_\downarrow(p)\epsilon_s(p) \tag{10}$$

$$\begin{aligned} E_2 = {} & \frac{N_a}{4\pi^2}\int_{-\pi}^{\pi}\mathrm{d}q \int_{-\pi}^{\pi}\mathrm{d}q'\, \delta_c(q)\delta_c(q')\tfrac{1}{2}f_{cc}(q,q') \\ & + \frac{N_a}{4\pi^2}\int_{-k_{F\uparrow}}^{k_{F\uparrow}}\mathrm{d}p \int_{-k_{F\uparrow}}^{k_{F\uparrow}}\mathrm{d}p'\, \delta_\downarrow(p)\delta_\downarrow(p')\tfrac{1}{2}f_{ss}(p,p') \\ & + \frac{N_a}{4\pi^2}\int_{-\pi}^{\pi}\mathrm{d}q \int_{-k_{F\uparrow}}^{k_{F\uparrow}}\mathrm{d}q\, \delta_c(q)\delta_\downarrow(p)f_{cs}(q,p)\,. \end{aligned} \tag{11}$$

As in Fermi liquid theory, the first and second functional derivatives of energy with respect to the fluctuations define the bands $\epsilon_c(q)$ and $\epsilon_s(p)$ of the pseudoparticles and their interactions, the f-functions $f_{cc}(q,q')$, $f_{ss}(p,p')$ and $f_{cs}(q,p)$, respectively. The latter are related to the pseudoparticle zero-momentum transfer forward scattering amplitudes [4]. Although the fluctuations of the right-hand sides of equations (10) and (11) are arbitrary in the sense that the expressions for the bands and f-functions are independent of them, only appropriate choices of $\delta_c(q)$ and $\delta_\downarrow(p)$ describe true eigenstates of the many-body system [4, 7]. The charge and spin pseudoparticle bands in the presence of a magnetic field can be expressed as [4]

$$\epsilon_c(q) = \int_Q^{K(q)} \mathrm{d}k'\, 2t\eta_c(k') \tag{12}$$

$$\epsilon_s(p) = \int_{B/u}^{S(p)} \mathrm{d}v'\, 2t\eta_s(v') \tag{13}$$

where $K(q)$ and $S(p)$ are the ground-state solutions of equations (4) and (5). We note that $\mu_c = \epsilon_c(2k_F) = 0$, $\mu_s = \epsilon_s(k_{F\downarrow}) = 0$. In the right-hand sides of equations (12) and (13) the distributions $2t\eta_c(k)$ and $2t\eta_s(v)$ are solutions of coupled integral equations of the same form as the Lieb and Wu equations except that the inhomogeneous term of the first of these equations is replaced by $2t\sin k$ (and $2\pi\sigma(\Lambda)$ by $(1/u)2t\eta_s(v)$, $v = \Lambda/u$) [3, 4]. The pseudoparticle velocities are defined as $v_c(q) = \mathrm{d}\epsilon_c(q)/\mathrm{d}q$ and $v_s(p) = \mathrm{d}\epsilon_s(p)/\mathrm{d}p$. In particular $v_c(2k_F)$ and $v_s(k_{F\downarrow})$ are the same velocities as the ones of [8, 9] (i.e. obey the same integral equations). A crucial advantage of our choice of variables q, p is that, given an eigenstate described by distributions (9), the corresponding crystal momentum P is always additive in the pseudoparticle pseudomomenta (see equation (2)). In fact, in contrast to the usual representation of Bethe *ansatz*, the back-flow effect only affects the energy bands $\epsilon_c(q)$ and $\epsilon_s(p)$ [4]. The charge band, equation (12), is such that its bandwidth $4t$, pseudo-Brillouin zone width 2π and pseudo-Fermi surface points $\pm 2k_F$ remain unaltered when varying H and u, which slightly change $\epsilon_c(q)$. In contrast to $\epsilon_c(q)$, the spin pseudoparticle band (13) is clearly affected by the magnetic field. In fact, the pseudo-Brillouin zone limits and Fermi surface points are given by $p = \pm k_{F\uparrow}$ and $p = \pm k_{F\downarrow}$, respectively. The effect of u (H) on the band $\epsilon_s(p)$, equation (13), is essentially to modify its bandwith (pseudo-Brillouin zone). On the other hand n affects both the bandwidth and the pseudo-Brillouin zone.

Except for spin singlet excitations involving complex roots (anti-bound states of down-spin pseudoparticles) [4], the elementary excitations can be described in terms of pseudoparticle–hole processes in the charge and spin bands. The full description of the low-lying excitations in the absolute ground state ($H = 0$ and $k_{F\downarrow} = k_F$) involves a 'frozen' up-spin pseudoparticle band which is always filled ('holes' are not allowed in the frozen band [4, 7]) and an upper 'conduction' charge band, $\epsilon_c^h(q) = U - \epsilon_c(q)$ [4, 7]. The spectra of the charge gapless [10] and across-gap [11] excitations, as well as of the triplet two-parametric excitations [10], can be written simply as

$$E_c = \epsilon_c(q_1) - \epsilon_c(q_0) \qquad P = q_1 - q_0 \qquad |q_0| < 2k_F \quad |q_1| > 2k_F \tag{14}$$

$$E_c^\Delta = \epsilon_c^h(q_1) - \epsilon_c(q_0) \qquad P = q_1 - q_0 \qquad |q_0| < 2k_F \tag{15}$$

$$E_t = -\epsilon_s(p_1) - \epsilon_s(p_0) \qquad P = 2k_F - p_1 - p_0 \qquad |p_0|, |p_1| < k_F \tag{16}$$

respectively, where the bands involved in the pseudoparticle–hole processes of the right-hand sides of equations (14)–(16) are given by equations (12) and (13) for the particular case of the absolute ground state ($B = \infty$). The excitations in the presence of a magnetic field are studied in [7].

As well as reducing the study of low lowing excitations to the usual language of band theory, the Landau–Luttinger approach allows the straightforward evaluation of the magnetic susceptibility (which involves the bands and f-functions) and low temperature specific heat: These can be readily obtained by replacing in the right-hand sides of equations (10) and (11) fluctuations $\delta_c(q)$ and $\delta_1(p)$ of appropriate form [4,7].

The f-functions of the right-hand side of equation (11), $f_{cc}(q,q')$, $f_{ss}(p,p')$, $f_{cs}(q,p)$ read [4]

$$\begin{aligned} f_{cc}(q,q') &= 2\pi v_c(q)\Phi_{cc}(q,q') + 2\pi v_c(q')\Phi_{cc}(q',q) \\ &\quad + [2\pi v_c(2k_F)] \sum_{j=\pm 1} \Phi_{cc}(2k_F j, q)\, \Phi_{cc}(2k_F j, q') \\ &\quad + [2\pi v_s(k_{F\downarrow})] \sum_{j=\pm 1} \Phi_{sc}(k_{F\downarrow} j, q)\, \Phi_{sc}(k_{F\downarrow} j, q') \end{aligned} \tag{17}$$

$$\begin{aligned} f_{ss}(p,p') &= 2\pi v_s(p)\Phi_{ss}(p,p') + 2\pi v_s(p')\Phi_{ss}(p',p) \\ &\quad + [2\pi v_s(k_{F\downarrow})] \sum_{j=\pm 1} \Phi_{ss}(k_{F\downarrow} j, p)\, \Phi_{ss}(k_{F\downarrow} j, p') \\ &\quad + [2\pi v_c(2k_F)] \sum_{j=\pm 1} \Phi_{cs}(2k_F j, p)\, \Phi_{cs}(2k_F j, p') \end{aligned} \tag{18}$$

$$\begin{aligned} f_{cs}(q,p) &= 2\pi v_c(q)\Phi_{cs}(q,p) + 2\pi v_s(p)\Phi_{sc}(p,q) \\ &\quad + [2\pi v_c(2k_F)] \sum_{j=\pm 1} \Phi_{cc}(2k_F j, q)\, \Phi_{cs}(2k_F j, p) \\ &\quad + [2\pi v_s(k_{F\downarrow})] \sum_{j=\pm 1} \Phi_{ss}(k_{F\downarrow} j, p)\, \Phi_{sc}(k_{F\downarrow} j, q)\,. \end{aligned} \tag{19}$$

In order to define the functions $\Phi_{cc}(q,q'), \Phi_{cs}(q,p), \Phi_{ss}(p,p'), \Phi_{sc}(p,q)$ appearing in the right-hand sides of equations (17)–(19), it is useful to introduce the auxiliary functions $\tilde{\Phi}_{cc}(k,k')$, $\tilde{\Phi}_{cs}(k,v)$, $\tilde{\Phi}_{ss}(v,v')$, $\tilde{\Phi}_{sc}(v,k)$ such that $\tilde{\Phi}_{cc}(K(q),K(q')) = \Phi_{cc}(q,q')$, $\tilde{\Phi}_{cs}(K(q),S(p)) = \Phi_{cs}(q,p)$, $\tilde{\Phi}_{ss}(S(p),S(p')) = \Phi_{ss}(p,p')$, $\tilde{\Phi}_{sc}(S(p),K(q)) = \Phi_{sc}(p,q)$, where $K(q)$, $S(p)$ are the ground-state solutions of equations (4) and (5). The auxiliary functions obey the following system of coupled integral equations [4]:

$$\tilde{\Phi}_{cc}(k,k') = \int_{-B/u}^{B/u} \mathrm{d}v'\, A_1^{(v')}(v',k)\tilde{\Phi}_{sc}(v',k') \tag{20}$$

$$\tilde{\Phi}_{cs}(k,v) = A_1(v,k) + \int_{-B/u}^{B/u} \mathrm{d}v'\, A_1^{(v')}(v',k)\tilde{\Phi}_{ss}(v',v) \tag{21}$$

$$\widetilde{\Phi}_{ss}(v,v') = A_2(v,v') - \int_{-Q}^{Q} \mathrm{d}k'\, A_1^{(k')}(v,k')\widetilde{\Phi}_{cs}(k',v') - \int_{-B/u}^{B/u} \mathrm{d}v''\, A_2^{(v'')}(v'',v)\widetilde{\Phi}_{ss}(v'',v') \tag{22}$$

$$\widetilde{\Phi}_{sc}(v,k) = -A_1(v,k) - \int_{-Q}^{Q} \mathrm{d}k' A_1^{(k')}(v,k')\widetilde{\Phi}_{cc}(k',k) - \int_{-B/u}^{B/u} \mathrm{d}v' A_2^{(v')}(v',v)\widetilde{\Phi}_{sc}(v',k) \tag{23}$$

where $A_1(v,k) = (1/\pi)\tan^{-1}(v - (1/u)\sin k)$, $A_2(v,v') = (1/\pi)\tan^{-1}\left(\frac{1}{2}(v-v')\right)$ and $A_1^{(k)}(v,k) = \mathrm{d}A_1(v,k)/\mathrm{d}k$, $A_1^{(v)}(v,k) = \mathrm{d}A_1(v,k)/\mathrm{d}v$, $A_2^{(v)}(v,v') = \mathrm{d}A_2(v,v')/\mathrm{d}v$.

The functions $\Phi_{cc}(q,q')$, $\Phi_{cs}(q,p)$, $\Phi_{ss}(p,p')$, $\Phi_{sc}(p,q)$ are the pseudoparticle renormalized scattering phase shifts [4]. In fact, the usual Bethe *ansatz* phase shifts, which for the 1D Hubbard model are four in number and can be evaluated by the method introduced by Korepin [12] for the massive Thirring model, may be written as a sum of two terms [4]. The first of these terms can be considered to be the scattering part of the phase shift. For the present model the scattering part of the four phase shifts are given by equations (20)–(23) [4]. The nature of the second term is discussed in [4].

The f-functions (17)–(19) regulate the electronic spectral properties of the model. In fact, the 'single-particle excitations' are described by fluctuations involving pairs of charge and spin pseudoparticles [4]. Each point (k,ω) of the two-dimensional space where the electronic spectral weight function is defined can be associated to one (or two) pair(s) of pseudoparticles [4]. On the other hand, there is a clear connection between the value of the electronic spectral function at a point (k,ω) and the interaction of the pair (or pairs) of pseudoparticles associated with that point [4]. Moreover, the interaction of the pseudoparticles determines the form of the electronic correlation functions. Particularly, the non-classical critical exponents which characterize the power law anomalies of the electronic momentum and the asymptotic behaviour of the correlation functions, are determined by the interaction of pseudoparticles with pseudo-momenta in the neighbourhood of the pseudo-Fermi points $q = \pm 2k_{\mathrm{F}}$, $p = \pm k_{\mathrm{F}\downarrow}$ [4]. These exponents can be derived by the conformal field approach [8, 9]. To illustrate the general character of the Landau–Luttinger liquid theory, we show that the finite size energy corrections and expressions for conformal dimensions of the fields in the former theory can be obtained by choosing particular forms for the fluctuations of the energy functional, equations (10) and (11), in the latter theory. For simplicity we restrict our considerations to the case when the number of electrons N and down-spin electrons M in the system remain unaltered [4]. We introduce the matrices $\mathbf{R}^{+}(q,p)$ and $\mathbf{R}^{-}(q,p)$ given by

$$\mathbf{R}^{\pm}(q,p) = \begin{pmatrix} \Phi_{cc}(q,\pm 2k_{\mathrm{F}}) & \Phi_{cs}(q,\pm k_{\mathrm{F}\downarrow}) \\ \Phi_{sc}(p,\pm 2k_{\mathrm{F}}) & \Phi_{ss}(p,\pm k_{\mathrm{F}\downarrow}) \end{pmatrix}. \tag{24}$$

These matrices describe the scattering of charge and spin pseudoparticles of arbitrary pseudo-momenta q, p, respectively, with right (+) and left (−) moving pseudoparticles of momenta at the pseudo-Fermi points.

It follows from equations (20) and (23) that the dressed charge matrix of [8,9] (here we use the definition of Waynarovich [9], which is the transpose of that of [8]) can be rewritten as

$$\mathbf{Z} = \mathbf{1} + \mathbf{R}^{+}(2k_{\mathrm{F}}, k_{\mathrm{F}\downarrow}) - \mathbf{R}^{-}(2k_{\mathrm{F}}, k_{\mathrm{F}\downarrow}) . \tag{25}$$

The form of equation (25) evidences that the matrix elements of $\mathbf{Z}$ are combinations of phase shifts associated with the scattering of pseudoparticles with pseudo-momenta at the pseudo-Fermi surfaces.

We consider now fluctuations $\delta_c(q) = M_c(q) - M_c^0(q)$, $\delta_\downarrow(p) = N_\downarrow(p) - N_\downarrow^0(p)$, such that

$$M_c(q) = \Theta\left(2k_{\mathrm{F}} + (\operatorname{sgn} q)q_c - |q|\right) + \frac{2\pi}{N_a}\left\{\sum_p [\delta(q - q_p^+) + \delta(q - q_p^-)]\right.$$
$$\left. - \sum_h [\delta(q - q_h^+) + \delta(q - q_h^-)]\right\} \tag{26}$$

$$N_\downarrow(p) = \Theta\left(k_{\mathrm{F}\downarrow} + (\operatorname{sgn} p)p_s - |p|\right) + \frac{2\pi}{N_a}\left\{\sum_p [\delta(p - p_p^+) + \delta(p - p_p^-)]\right.$$
$$\left. - \sum_h [\delta(p - p_h^+) + \delta(p - p_h^-)]\right\} \tag{27}$$

where $|q_c|$, $|2k_{\mathrm{F}} \mp q_p^{\pm}|$, , $|2k_{\mathrm{F}} \mp q_h^{\pm}| \ll 2k_{\mathrm{F}}$ and $|p_s|$, $|k_{\mathrm{F}\downarrow} \mp p_p^{\pm}|$, $|k_{\mathrm{F}\downarrow} \mp p_h^{\pm}| \ll k_{\mathrm{F}\downarrow}$. The first term of the right-hand side of equation (26) ((27)) includes charge (spin) pseudoparticle–hole processes from pseudo-momenta close to $-2k_{\mathrm{F}}(-k_{\mathrm{F}\downarrow})$ to pseudo-momenta in the neighbourhood of $2k_{\mathrm{F}}(k_{\mathrm{F}\downarrow})$. $D_c = (N_a/2\pi)\,q_c$ ($D_s = (N_a/2\pi)\,q_s$) gives the number of pseudoparticles transferred ($D_c \ll N$, $D_s \ll M$). On the other hand, the second term of the right-hand side of equation (26) ((27)) describes charge (spin) pseudoparticle–hole processes around the points $\pm 2k_{\mathrm{F}}$ ($\pm k_{\mathrm{F}\downarrow}$). The indices p and h refer to particle and hole summations. + and − refer to right- and left-moving pseudoparticles. The asymptotic behaviour of the correlation functions is determined by these pseudoparticle–hole processes which involve exclusively pseudo-momenta in the neighbourhood of the pseudo-Fermi points.

We define the numbers

$$N_c^{\pm} = \pm\frac{N_a}{2\pi}\left(\sum_p q_p^{\pm} - \sum_h q_h^{\pm}\right)$$
$$N_s^{\pm} = \pm\frac{N_a}{2\pi}\left(\sum_p p_p^{\pm} - \sum_h p_h^{\pm}\right). \tag{28}$$

To evaluate the integrals of the right-hand sides of the energies (10) and (11) we expand $\epsilon_c(q)$ and $\epsilon_\downarrow(p)$ around the pseudo-Fermi points. Moreover to the two leading orders only the f-functions connecting pseudomomenta at the pseudo-Fermi points give contributions to the energy corrections. The energy and momentum associated

with the distributions (26) and (27) are given by [4]

$$\begin{aligned} EN_a = E_0 N_a &+ \left(2\pi v_c(2k_F) + \sum_{j=\pm 1}(j) f_{cc}(2k_F, 2k_F j)\right) D_c^2 \\ &+ \left(2\pi v_s(k_{F\downarrow}) + \sum_{j=\pm 1}(j) f_{ss}(k_{F\downarrow}, k_{F\downarrow} j)\right) D_s^2 \\ &+ \left(\sum_{j=\pm 1}(j) f_{cs}(2k_F, k_{F\downarrow} j)\right) 2 D_c D_s \\ &+ 2\pi v_c(2k_F)[N_c^+ + N_c^-] + 2\pi v_s(k_{F\downarrow})[N_s^+ + N_s^-] \end{aligned} \tag{29}$$

$$P = \frac{2\pi}{N_a}[N D_c + M D_s + N_c^+ - N_c^- + N_s^+ - N_s^-]. \tag{30}$$

The use of equations (17)–(19), (24) and (25) allows us to rewrite the energy (29) as follows:

$$E = E_0 + \frac{2\pi}{N_a}[v_c(2k_F)(\Delta_c^+ + \Delta_c^-) + v_s(k_{F\downarrow})(\Delta_s^+ + \Delta_s^-)] \tag{31}$$

where

$$\Delta_c^{\pm} = \tfrac{1}{2}(\xi_{cc} D_c + \xi_{cs} D_s)^2 + N_c^{\pm} \qquad \Delta_s^{\pm} = \tfrac{1}{2}(\xi_{sc} D_c + \xi_{ss} D_s)^2 + N_s^{\pm} \tag{32}$$

and ξ_{cc}, ξ_{cs}, ξ_{sc} and ξ_{ss} are the elements of matrix (25). As our fluctuations refer to the case when $\Delta N = \Delta M = 0$ (N and M remain unaltered), equations (30) and (31) and (32) are precisely the momentum, finite-size energy correction and conformal field dimensions $\Delta_c^{\pm}$, $\Delta_s^{\pm}$, respectively, of [8,9]. The leading term in the asymptotic expansion of the correlation functions decays with critical exponents obtained from (32) by minimizing with respect to D_c, D_s (i.e., by minimizing with respect to $\delta_c(q)$, $\delta_1(p)$) [4, 8, 9]. When $\Delta N \neq 0$ or $\Delta M \neq 0$, extra energy boundary terms appear in the right-hand sides of equations (29) and (32) [4, 8, 9]. These terms are functions purely of ΔN, ΔM and of the renormalized pseudoparticle phase shifts.

It follows from the present results that the critical exponents of the model are fully determined by the pseudoparticle renormalized interactions [4]. In fact, these exponents are exclusively functions of renormalized scattering phase shifts associated to the pseudoparticle forward scattering processes such that both pseudo-momenta are pseudo-Fermi points. On the other hand, the Landau–Luttinger liquid formulation introduces a more general framework: it contains full information about the pseudoparticle renormalized interactions for any pair of pseudo-momenta [4].

In this letter we have introduced the concept of a Landau–Luttinger liquid. In addition to clarifying the physics by reducing the study of the low-lying excitations to the familiar language of band theory, the formulation used here allows explicit calculation of the f-functions which, as in Fermi liquid theory, are related to the forward scattering amplitudes of the pseudoparticles. Moreover, our results show that the renormalized Landau–Luttinger theory has only forward scattering. Although the formal similarities with the Fermi liquid theory are striking, we would like to stress the crucial differences with the latter. It is important to realize that in the former the pseudoparticles involved in the description of the low-energy properties refer to

exact eigenstates of the many-body system. This is in contrast to Fermi liquid theory where the quasiparticles describe approximate eigenstates of finite lifetime near the Fermi surface. Finally, there is a second important difference which we believe to be common to all non-trivial higher dimensional fermionic liquids for which the overlap integral of [2] vanishes ('infrared catastrophe'): there is no one-to-one correspondence between the pseudoparticles of such non-trivial liquids (including the present Landau–Luttinger liquids) and the real particles (electrons) upon turning off adiabatically the bare interaction. This is obviously due to the fact that in these non-trivial liquids the usual Fermi liquid fixed point is excluded.

The present results may offer insight into the physics of higher dimensional systems [2] where, in contrast to the 1D case, the renormalized interaction of the new Landau liquid pseudoparticles could eventually produce bound states, providing a mechanism for high-T_c superconductivity. Moreover, we believe they are relevant to quasi-one-dimensional materials. For example coupling the charge and spin pseudoparticles to $4k_F$ and $2k_F$ phonon modes [4], respectively, results in $4k_F$ (charge) and $2k_F$ (spin) instabilities for large and intermediate on-site U, in agreement with experiment [13].

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